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Determining tooth thickness of various gear types

How to calculate the nominal values of chordal tooth thickness in all types of gearing.

In order to determine the tooth size of a gear after taking into account the backlash allowance, you first must determine what the nominal tooth thickness should be. There are three methods for determining this value: chordal tooth thickness measurement, span measurement, and over pin or ball measurement. For this article, we will discuss chordal tooth thickness measurements.

The chordal tooth thickness measurement method uses a tooth caliper that is referenced from the gear's tip diameter. The thickness is measured at the reference circle as detailed in Figure 1.

For spur gears, the formulas to calculate the chordal tooth thickness are detailed in Table 1.

For rack, regardless if they are straight tooth or helical tooth, the formulas are simplified because the gear tooth profile is trapezoid. These formulas are detailed in Table 2.

Measuring helical gears can be done in either the normal plane or the transverse plane dependent on how the gears are cut. The formulas for measurements in the normal plane are detailed in Table 3 and those for measurements in the transverse plane are detailed in Table 4.

The measurement of straight tooth bevel gearing is dependent on the tooth system. The formulas for measurements in the standard system are detailed in Table 5 and the formulas for measurements for bevel gears cut using the Gleason system are detailed in Table 6.

If a straight bevel gear is cut by a Gleason straight bevel cutter, the tooth angle should be adjusted according to:

$$\text{Tooth angle } (^\circ) = \frac{180}{\pi R}\left(\frac{s}{2} + h_f \tan\alpha\right)$$

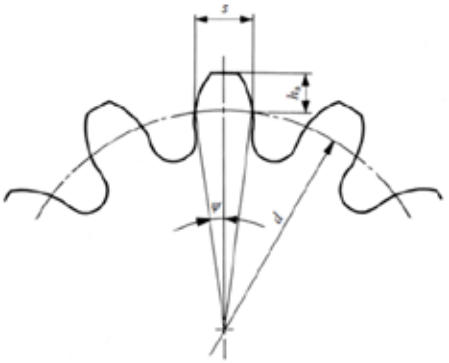

Figure 1: Chordal tooth thickness method.

This angle is used as a reference in determining the tooth thickness, s, when setting up the gear cutting machine.

For spiral bevel gears cut using the Gleason system, the formulas to calculate the chordal tooth thickness are detailed in Table 7.

The calculations of chordal tooth thickness of a Gleason spiral bevel gear are so complicated that I will not go any further in this presentation.

The measuring of worm gearing can be done in either the axial plane or the normal plane dependent on how the gears are cut. The formulas for measurements in the axial plane are detailed in Table 8, and those for measurements in the normal plane are detailed in Table 9.

Using these tables and formulas, you will be able to determine the proper chordal tooth thickness for your gear. From these values and the measured values, you can determine the tooth thinning or backlash allowance cut into the gear.

No.	Item	Symbol	Formula	Example
1	Tooth thickness	s	$\left(\frac{\pi}{2} + 2x\tan\alpha\right)m$	$m = 10$ $\alpha = 20^\circ$ $z = 12$ $x = +0.3$ $h_a = 13.000$ $s = 17.8918$ $\psi = 8.54270^\circ$ $\bar{s} = 17.8256$ $\bar{h}_a = 13.6657$
2	Tooth thickness half angle	ψ	$\frac{90}{z} + \frac{360x\tan\alpha}{\pi z}$	
3	Chordal tooth thickness	$\bar{s}$	$zm\sin\psi$	
4	Chordal height	$\bar{h}_a$	$\frac{zm}{2}(1-\cos\psi) + h_a$	

Table 1: Equations for spur gear chordal tooth thickness.

No.	Item	Symbol	Formula	Example
1	Chordal tooth thickness	$\bar{s}$	$\frac{\pi m}{2}$ or $\frac{\pi m_n}{2}$	$m = 3$ $\alpha = 20^\circ$ $\bar{s} = 4.7124$ $h_a = 3.0000$
2	Chordal height	$\bar{h}_a$	h_a	

Table 2: Chordal tooth thickness of racks.

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness	s_n	$\left(\frac{\pi}{2} + 2x_n\tan\alpha_n\right)m_n$	$m_n = 5$ $\alpha_n = 20^\circ$ $\beta = 25^\circ 00' 00''$ $z = 16$ $x_n = +0.2$ $h_a = 6.0000$ $s_n = 8.5819$ $z_v = 21.4928$ $\psi_v = 4.57556^\circ$ $\bar{s} = 8.5728$ $\bar{h}_a = 6.1712$
2	Number of teeth of an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Tooth thickness half angle	ψ_v	$\frac{90}{z_v} + \frac{360x_n\tan\alpha_n}{\pi z_v}$	
4	Chordal tooth thickness	$\bar{s}$	$z_v m_n \sin\psi_v$	
5	Chordal height	$\bar{h}_a$	$\frac{z_v m_n}{2}(1-\cos\psi_v) + h_a$	

Table 3: Equations for chordal tooth thickness of helical gears in the normal system.

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness	s_n	$\left(\frac{\pi}{2} + 2x_t\tan\alpha_t\right)m_t\cos\beta$	$m_t = 2.5$ $\alpha_t = 20^\circ$ $\beta = 21^\circ 30' 00''$ $z = 20$ $x_t = 0$ $h_a = 2.5$ $s_n = 3.6537$ $z_v = 24.8311$ $\psi_v = 3.62448^\circ$ $\bar{s} = 3.6513$ $\bar{h}_a = 2.5578$
2	Number of teeth in an equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	
3	Tooth thickness half angle	ψ_v	$\frac{90}{z_v} + \frac{360x_t\tan\alpha_t}{\pi z_v}$	
4	Chordal tooth thickness	$\bar{s}$	$z_v m_t\cos\beta\sin\psi_v$	
5	Chordal height	$\bar{h}_a$	$\frac{z_v m_t\cos\beta}{2}(1-\cos\psi_v) + h_a$	

Table 4: Equations for chordal tooth thickness of helical gears in the transverse system.

No.	Item	Symbol	Formula	Example
1	Tooth thickness	s	$\frac{\pi m}{2}$	$m = 4$ $\alpha = 20°$ $\Sigma = 90°$ $z_1 = 16$ $z_2 = 40$ $d_1 = 64$ $d_2 = 160$ $h_a = 4.0000$ $\delta_1 = 21.8014°$ $\delta_2 = 68.1986°$ $s = 6.2832$ $z_{v1} = 17.2325$ $z_{v2} = 107.7033$ $R_{v1} = 34.4650$ $R_{v2} = 215.4066$ $\psi_{v1} = 5.2227°$ $\psi_{v2} = 0.83563°$ $\bar{s}_1 = 6.2745$ $\bar{s}_2 = 6.2830$ $\bar{h}_{a1} = 4.1431$ $\bar{h}_{a2} = 4.0229$
2	Number of teeth of an equivalent spur gear	z_v	$\frac{z}{\cos\delta}$	
3	Back cone distance	R_v	$\frac{d}{2\cos\delta}$	
4	Tooth thickness half angle	ψ_v	$\frac{90}{z_v}$	
5	Chordal tooth thickness	$\bar{s}$	$z_v m \sin\psi_v$	
6	Chordal height	$\bar{h}_a$	$h_a + R_v(1-\cos\psi_v)$	

Table 5: Equations for chordal tooth thickness of standard straight bevel gears.

No.	Item	Symbol	Formula	Example
1	Tooth thickness factor (Coefficient of horizontal profile shift)	K	Obtain from Figure 2	$m = 4$ $\alpha = 20°$ $\Sigma = 90°$ $z_1 = 16$ $z_2 = 40$ $z_1/z_2 = 0.4$ $K = 0.0259$ $h_{a1} = 5.5456$ $h_{a2} = 2.4544$ $\delta_1 = 21.8014°$ $\delta_2 = 68.1986°$ $s_1 = 7.5119$ $s_2 = 5.0545$ $\bar{s}_1 = 7.4946$ $\bar{s}_2 = 5.0536$ $\bar{h}_{a1} = 5.7502$ $\bar{h}_{a2} = 2.4692$
2	Tooth thickness	s_1 s_2	$\pi m - s_2$ $\frac{\pi m}{2} - (h_{a1} - h_{a2})\tan\alpha - Km$	
3	Chordal tooth thickness	$\bar{s}$	$s - \frac{s^3}{6d^2}$	
4	Chordal height	$\bar{h}_a$	$h_a + \frac{s^3\cos\delta}{4d}$	

Table 6: Equations for chordal tooth thickness of Gleason straight bevel gears.

No.	Item	Symbol	Formula	Example
1	Tooth thickness factor	K	Obtain from Figure 3	$\Sigma = 90°$ $m = 3$ $\alpha_n = 20°$ $z_1 = 20$ $z_2 = 40$ $\beta_m = 35°$ $h_{a1} = 3.4275$ $h_{a2} = 1.6725$ $K = 0.060$ $p = 9.4248$ $s_1 = 5.6722$ $s_2 = 3.7526$
2	Tooth thickness	s_1 s_2	$p - s_2$ $\frac{p}{2} - (h_{a1} - h_{a2})\frac{\tan\alpha_n}{\cos\beta_m} - Km$	

Table 7: Equations for chordal tooth thickness of Gleason spiral bevel gears.

No.	Item	Symbol	Formula	Example
1	Axial tooth thickness of worm	s_{x1}	$\frac{\pi m_x}{2}$	$m_x = 3$ $m_t = 3$ $\alpha_n = 20°$ $z_1 = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 90$ $a = 65$ $x_{t2} = +0.33333$ $h_{a1} = 3.0000$ $h_{a2} = 4.0000$ $\gamma = 8.97263°$ $\alpha_t = 20.22780°$ $s_{x1} = 4.71239$ $s_{t2} = 5.44934$ $z_{v2} = 31.12885$ $\psi_{v2} = 3.34335°$ $\bar{s}_1 = 4.6547$ $\bar{s}_2 = 5.3796$ $\bar{h}_{a1} = 3.0035$ $\bar{h}_{a2} = 4.0785$
	Transverse tooth thickness of worm wheel	s_{t2}	$\left(\frac{\pi}{2} + 2x_{t2}\tan\alpha_t\right)m_t$	
2	No. of teeth in an equivalent spur gear (Worm wheel)	z_{v2}	$\frac{z_2}{\cos^3\gamma}$	
3	Tooth thickness half angle (Worm wheel)	ψ_{v2}	$\frac{90}{z_{v2}} + \frac{360\,x_{t2}\tan\alpha_t}{\pi z_{v2}}$	
4	Chordal tooth thickness	$\bar{s}_1$ $\bar{s}_2$	$s_{x1}\cos\gamma$ $z_{v2}m_t\cos\gamma\sin\psi_{v2}$	
5	Chordal height	$\bar{h}_{a1}$	$h_{a1} + \frac{(s_{x1}\sin\gamma\cos\gamma)^2}{4d_1}$	
		$\bar{h}_{a2}$	$h_{a2} + \frac{z_{v2}m_t\cos\gamma}{2}(1-\cos\psi_{v2})$	

Table 8: Equations for chordal tooth thickness of an axial module worm gear pair.

No.	Item	Symbol	Formula	Example
1	Normal tooth thickness of worm	s_{n1}	$\frac{\pi m_n}{2}$	$m_n = 3$ $\alpha_n = 20°$ $z_1 = 2$ $z_2 = 30$ $d_1 = 38$ $d_2 = 91.1433$ $a = 65$ $x_{n2} = 0.14278$ $h_{a1} = 3.0000$ $h_{a2} = 3.42835$ $\gamma = 9.08472°$ $s_{n1} = 4.71239$ $s_{n2} = 5.02419$ $z_{v2} = 31.15789$ $\psi_{v2} = 3.07964°$ $\bar{s}_1 = 4.7124$ $\bar{s}_2 = 5.0218$ $\bar{h}_{a1} = 3.0036$ $\bar{h}_{a2} = 3.4958$
	Transverse tooth thickness of worm wheel	s_{n2}	$\left(\frac{\pi}{2} + 2x_{n2}\tan\alpha_n\right)m_n$	
2	No. of teeth in an equivalent spur gear (Worm wheel)	z_{v2}	$\frac{z_2}{\cos^3\gamma}$	
3	Tooth thickness half angle (Worm wheel)	ψ_{v2}	$\frac{90}{z_{v2}} + \frac{360\,x_{n2}\tan\alpha_n}{\pi z_{v2}}$	
4	Chordal tooth thickness	$\bar{s}_1$ $\bar{s}_2$	s_{n1} $z_{v2}m_n\sin\psi_{v2}$	
5	Chordal height	$\bar{h}_{a1}$	$h_{a1} + \frac{(s_{n1}\sin\gamma)^2}{4d_1}$	
		$\bar{h}_{a2}$	$h_{a2} + \frac{z_{v2}m_n}{2}(1-\cos\psi_{v2})$	

Table 9: Equations for chordal tooth thickness of a normal module worm gear pair.

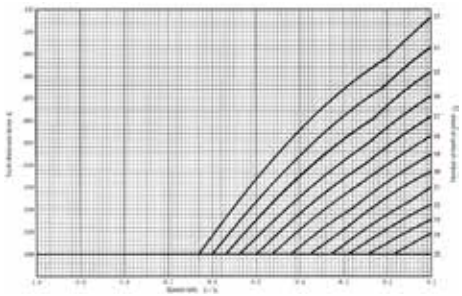

Figure 2: Chart to determine the tooth thickness factor k for Gleason straight bevel gear.

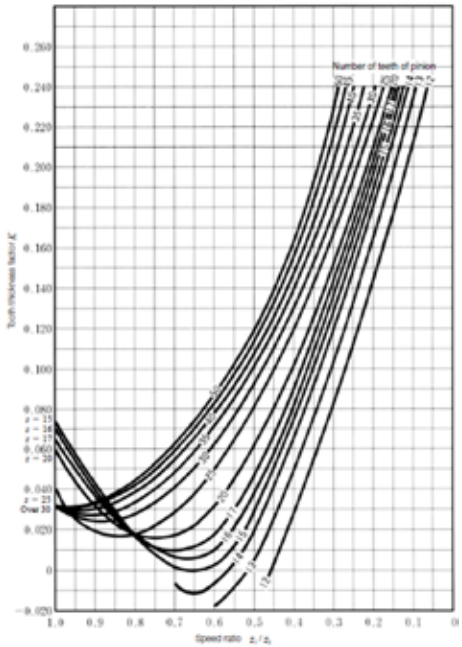

Figure 3: Chart to determine the tooth thickness factor k for Gleason spiral bevel gears.

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